Efficient Parallel Numerical Analysis of Rotating Bodies based on Hierarchical Domain Decomposition Method

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This paper deals with three-dimensional non-steady eddy current analysis of a rotating machine. In general, high efficiency in parallel computing with a moving body is difficult to achieve. The hierarchical domain decomposition method (HDDM) is known as an efficient parallel finite element method. However, in cases that involve a moving body, the HDDM with static domain decomposition has not attained sufficient parallel efficiency. Moreover, the cost of dynamic domain decomposition is substantial. In this paper, we propose a new domain decomposition technique for the HDDM that enables us to achieve efficient scalability on massively parallel computers. Our method's parallel efficiency was 93.3% on 96 nodes (1,536 cores) of the Oakleaf-FX supercomputer. Furthermore, an induction motor model with a seven million degrees of freedom mesh whose solution by conventional sequential computation requires more than a month was successfully solved in approximately 1.60 hours using the proposed method.

Index Terms—**Domain decomposition method, Finite element analysis, Parallel computing, Rotating machines**

I. Introduction

ROTATING machines such as an electric generator or motors is representative. Such devices have become essential in our lives; thus, the development of an efficient rotating machine has become necessary to reduce cost and environmental loading. Designing rotating machines includes electromagnetic analysis, which takes substantial time; thus, methods for reducing the time steps, such as time periodic explicit error correction (TP-EEC) [1]-[2] and time differential correction (TDC) [3], and for reducing the computation time, such as the parallel iterative method [4]-[5], have been proposed. However, these methods cannot utilize massively parallel computers efficiently, and still take substantial time to analyze a rotating machine. However, for non-linear magnetostatic problems, time-harmonic eddy current problems, and high-frequency electromagnetic problems, problems with billions of degrees of freedom (DOFs) have been solved efficiently on the massively parallel computers using the hierarchical domain decomposition method (HDDM) [6]-[8]. Nevertheless, in cases that involve a moving body, the HDDM with static domain decomposition has not attained sufficient parallel efficiency. Moreover, the cost of dynamic domain decomposition is substantial. In this paper, we propose a new domain decomposition technique for the HDDM for analyzing devices including moving bodies efficiently on massively parallel computers. Then, we confirm the computational efficiency of our proposed method through an analysis of a simplified induction motor model.

II. Proposed Method

In the HDDM, the analysis domain is decomposed into non-overlapping subdomains, and then the unknowns are partitioned to the interior of the subdomains and the interface boundary between subdomains. Further, the Schur complement equation derived from the original linear equation by the static condensation onto the interface boundary is solved in the parallel environment.

To treat the model that includes moving bodies efficiently on massively parallel environments, meshes of stationary and moving bodies are decomposed independently. The connection surface where bodies come in contact with each other appears on the surface of the subdomains; thus, the connection surface is treated as the interface boundary shared between subdomains. Here, a communication table relating to the original interface boundary and communication tables relating to the connection surface changing with time evolution are prepared separately. Then, they are used in combination at each time step.

III. Numerical Examples

A. Model

The simplified induction motor model is analyzed as a numerical example. The mesh of stationary body has 2,903,040 elements and 4,242,912 nodes, that of moving body has 2,808,960 elements and 4,101,120 nodes. Total DOFs are 7,395,232. In this paper, we use two models. One is a model combining the meshes of the stationary and moving bodies (Not moving). The other is a model decomposing the meshes of the stationary and moving bodies independently; thus, the moving body actually moves (Moving). Then, we compare the elapsed time.

The HDDM is implemented by hybrid parallelization. In each computer node, one MPI process works, and each MPI process starts as many OpenMP threads as cores in each computer node.

B. Strong scaling tests

To measure the parallel performance, strong scaling tests are run on an Oakleaf-FX supercomputer [9]. The Oakleaf-FX super computer consists of 4,800 computer nodes of a Fujitsu PRIMEHPC FX10. Each computer node has a CPU with 16

TABLE I
NUMBERS OF PARTS AND SUBDOMAINS

Nodes (cores)	Not moving part × subdomains	Moving part × sugdomains (Upper: Stationary body Lower: Moving body)
6 (96)	6 × 9,600	3 × 9,600 3 × 9,600
12 (192)	12 × 4,800	6 × 4,800 6 × 4,800
24 (384)	24 × 2,400	12 × 2,400 12 × 2,400
48 (768)	48 × 1,200	24 × 1,200 24 × 1,200
96 (1,536)	96 × 600	48 × 600 48 × 600
192 (3,072)	192 × 300	96 × 300 96 × 300
384 (6,144)	384 × 150	192 × 150 192 × 150

cores and 32GB memory. Domain decompositions are performed to ensure that the total number of subdomains is 57,600 (TABLE I).

The Conjugate Gradient (CG) method with the simplified block diagonal scaling is applied to solve the Schur complement equation of the HDDM. In each subdomain, the CG method is used as the solver and its convergence criteria is set to 10^{-9}. A shifted incomplete Cholesky factorization is used as the preconditioner with the accelerative parameter 1.2. In order to fix the total computation amount and observe whether the elapsed time becomes shorter depending on the number of nodes, the CG method for the Schur complement equation is stopped after 100 iterations, and 100 time steps are performed.

TABLE II shows elapsed times and parallel efficiencies. Fig. 1 shows speed-up ratios. Parallel efficiencies are over 90% up to 96 nodes. Furthermore, speed-up ratios are ideal up to 96 nodes, too. It is confirmed that the parallel computing with moving body is performed without lowering the efficiency. Moreover, the elapsed time of “Moving” increases by approximately 15%.

C. Comparison with sequential computation

In order to compare the computation time with conventional sequential computation, NEXST_Magnetic [10] is executed using one core of Oakleaf-FX. The simplified induction motor model is analyzed 1,200 time steps. As a result, the sequential computation takes 3.49 hours for five time steps. On the other hand, the proposed method using 48 nodes takes only approximately 2 minutes for five time steps, and 8.39 hours for 1,200 time steps. Therefore, it is guessed that the sequential computation takes over one month for 1,200 time steps. Furthermore, its parallel efficiency falls, but the proposed method using 384 nodes takes only approximately 1.60 hours for 1,200 time steps.

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TABLE II
ELAPSED TIMES AND PARALLEL EFFICIENCIES

Nodes	Not moving		Moving		Ratio [%] (Moving / Not moving)
	Time [s]	Parallel efficiency [%]	Time [s]	Parallel efficiency [%]	
6	7,315	-	8,520	-	116.5
12	3,637	101.0	4,231	101.3	116.3
24	1,836	99.5	2,078	103.2	113.2
48	950	95.8	1,072	99.2	112.9
96	500	90.9	568	93.3	113.6
192	300	74.7	330	80.3	108.6
384	194	58.3	248	53.0	127.9

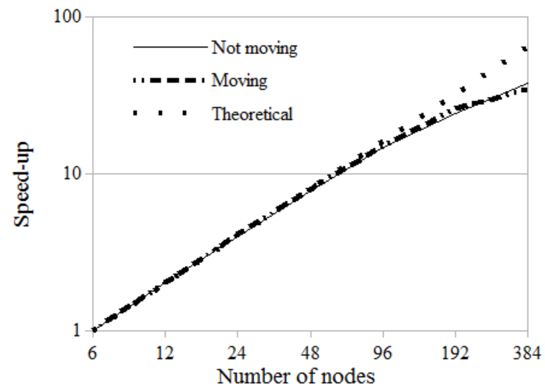

Fig. 1. Speed-up ratios of strong scaling.

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